Analysis of the Optimized Schwarz Method with a Coarse Grid Correction

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A popular method for solving large boundary value problems on parallel computers is the domain decomposition approach. Given a domain Ω, one first picks a domain decomposition $\Omega = \Omega_1 \cup \ldots \cup \Omega_p$, and then one solves boundary value problems on each subdomain in parallel. The local solutions do not glue together into a global solution, but if we iterate the process, one can usually obtain convergence. In the Schwarz method, the PDE for each subdomain Ω_j is the same as the one for Ω, and the boundary data on $\partial\Omega_j$ is Dirichlet data obtained from the local solutions on the subdomains adjacent to Ω_j. The convergence rate ρ depends chiefly on p, the number of subdomains. To obtain an algorithm which converges at a rate ρ which does not depend on the number of subdomains, it is customary to introduce a coarse-grid correction. The Optimized Schwarz Method is a Schwarz method that uses Robin data on the artificial interfaces $\partial\Omega_j$. The analysis of these methods is very challenging, and there is currently no published analysis of the Optimized Schwarz Method with a coarse grid correction. We will provide such an analysis and give the optimized Robin parameter which leads to the best possible convergence rate.

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